Math 1321 Week 10 Lab Mid-term Review

1. Let $f(x, y, z) = z + \sin \frac{z}{y} \ln(x^2 - xy + y^2)$.

   (a) Find the partial derivatives f_x, f_y and f_z.

   (b) Find the linear approximation of f nearby the point $(1, 1, \pi/2)$ and estimate value of $f(1.01, 1.02, \pi/2 + 0.03)$.

2. Let $z = f(x, y)$ be the function implicitly defined by the equation $e^z + z + xy = 3$.

(a) Find the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ [Hint: use Implicit Function Theorem].

(b) First verify that $z = 0$ when $x = 2$ and $y = 1$. Then find the linear approximation of $z = f(x, y)$ nearby the point $(2, 1)$.

3. Let D be a closed bounded set in xOy plane defined by $\{(x, y) \in \mathbb{R}^2 \mid x^2 - 4 \leq y \leq 4 - x^2\}$ and $f(x, y) = x^2 + y^2 - 6y + 4$ Find the maximum and minimum value of f on D.

4. Let R be the rectangular region $D = [0,1] \times [0,2] = \{(x,y) \in \mathbb{R}^2 \mid 0 \le x \le 1,\ 0 \le y \le 2\}$. Estimate the integral $\iint_R \ln(x^2 + y^2 + 1)\, dxdy$ using double Riemann sum. Divide R into 8 0.5 by 0.5 squares and choose the sample point to be the upper right corner of each square.