Name: Score:

Math 1321 Week 11 Lab Due Thursday 11/20

1. **(2 points)** Sketch the region of integration and interchange the order and evaluate.

$$\int_0^1 \int_{1-y}^1 xy^2 dxdy$$

2. **Polar Coordinates**

(a) **(2 points)** Evaluate $\int\int_{D_a} e^{-(x^2+y^2)}dxdy$ where D_a is the disk $x^2 + y^2 \leq a$.

(b) **(1 points)** Show that the limit of the integral as $a \to \infty$ is π.

3. **Gaussian Integral**

(a) **(2 point)** There is no direct way to compute the following integral using x, y coordinates.

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Yes, believe it or not, the answer is $\sqrt{\pi}$! In fact, prove it. Hint: Use your answers to the previous question and *don't* use polar coordinates this time.

(By the way, the $\int_{-\infty}^{\infty} e^{-x^2} dx$ is called the Gaussian Integral and it plays a role in modern probability theory and quantum mechanics.)

(b) **(1 point)** Evaluate the integral $\int_0^{\infty} e^{-x^2} dx$