Math 1321 Week 13 Lab Worksheet

1. **Line Integral & Green's Theorem** Find the work which is done by the force field $(3x + 4y)\mathbf{i} + (8x + 9y)\mathbf{j}$ on a particle that moves once around the ellipse $4x^2 + 9y^2 = 36$ by

   (a) Directly evaluating the line integral.

   (b) Using Green's Theorem to evaluate the line integral.

2. Evaluate the integral $\int_C (e^x \sin y - 2y)\,dx + (e^x \cos y - 2)\,dy$, where C is the boundary of the region bounded by $y = \sin x, 0 \leq x \leq \pi$, and the x-axis.

3. Let $\mathbf{F}$ be the vector field $(x + \sin^2 y)\mathbf{i} + (x^2 - y)\mathbf{j}$. Evaluate the integral $\int_C \mathbf{F} \cdot \mathbf{n}\,ds$, where C is the boundary of the upper half of the disk $(x-1)^2 + y^2 \leq 1$ and $\mathbf{n}$ is the unit normal vector to C.

4. Find the area of the region bounded by the x-axis and one segment of the cycloid $x = t - \sin t,\ y = 1 - \cos t$ where $0 \leq t \leq 2\pi$.