Name: Score: /8

Math 1321 Week 8 Lab Worksheet Due Thursday 10/30

1. **Partial Derivatives:**

(a) (**1 point**) Suppose $z = (1 + xy)^y$. Find first partial derivatives $\dfrac{\partial z}{\partial x}$, $\dfrac{\partial z}{\partial y}$.

(b) (**1 point**) Suppose $z = \arctan \dfrac{y}{x}$. Find second partial derivatives $\dfrac{\partial^2 z}{\partial x^2}$, $\dfrac{\partial^2 z}{\partial x \partial y}$, $\dfrac{\partial^2 z}{\partial y^2}$.

(c) (**1 point**) Show that $u = z \arctan \dfrac{x}{y}$ satisfies the Laplace's equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$

2. **Directional Derivatives and Gradient Vector:**

Suppose $f(x, y) = x^2 - xy + y^2$.

(a) (**1 point**) Find all the directional derivatives at $(1, 1)$, i.e. $\mathbf{D_u} f(1, 1)$, with $u = (\cos\alpha, \sin\alpha)$.

(b) (**1 point**) When does the directional derivative get to its maximum? its minimum? When is the directional derivative 0?

3. **Wind-Chill** The wind-chill index W is the perceived temperature when the actual temperature is T and the wind speed is v so, we can write $W = f(T, v)$.

   (a) (**2 points**) The following table of values is an excerpt from Table 1 in Section 11.1. Use the table to find a linear approximation to the wind-chill index function when T is near -15^oC and v is near $50km/h$. [Hint: Analogous to the procedure similar to Table 1 in Section 11.3 of your textbook, use the following table to find the partial derivatives f_T and f_v first.]

Wind speed (km/h)

Actual temperature (°C) T \ v	20	30	40	50	60	70
−10	−18	−20	−21	−22	−23	−23
−15	−24	−26	−27	−29	−30	−30
−20	−30	−33	−34	−35	−36	−37
−25	−37	−39	−41	−42	−43	−44

   (b) (**1 point**) Estimate the wind-chill index when the temperature is -17^oC and the wind speed is $55km/h$.