Name: Score:

Math 1321 Week 3 Worksheet Due Thursday 09/18

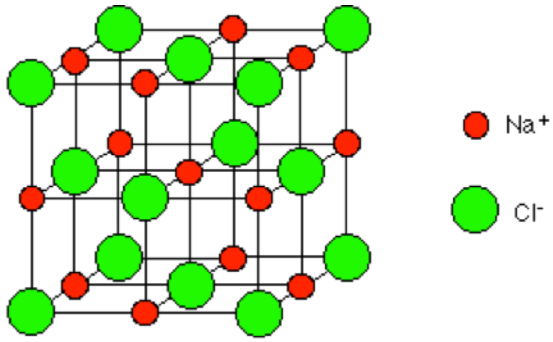
1. (a) Compute the first three terms (whether zero or not) of Taylor Series for $f(x) = \sin(2x)$ centered about $x = \pi$.

   (b) Write down the n-th term in the Taylor series.

   (c) On a separate sheet plot the degree 0, 1, 2, and 3 approximations on the same plot over the interval $[0, 2\pi]$.

   (d) Use Taylor's Inequality to find the optimal bound of the remainder $R_3(x)$ over the interval $\text{I} = [0, 2\pi]$.

2. 3D Crystals of Na^+ and Cl^- form a square lattice like the one shown below.



The energy required to *pull* apart all the ions in the crystal from the central Na^+ ion can be approximated by the sum of the work $W(x)$ required to break each bond as a function of the distances x relative to the central Na^+ atom. This results in an alternating series. Focusing on the Na^+ ion at the center of the cubic unit cell, note that this positive ion is attracted to 6 Cl^- ions at distance r^*, repelled by 12 Na^+ ions at distance $\sqrt{2}r^*$ away, then attracted to 8 Cl- ions at distance $\sqrt{3}r^*$ away, and so on out to the edge of the crystal.

$$
\begin{aligned}
E_{Na^+} =& \frac{C}{r^*}\Big(\frac{6}{1} - \frac{12}{\sqrt{2}} + \frac{8}{\sqrt{3}} - \frac{6}{\sqrt{4}} + \frac{24}{\sqrt{5}} + \dots\Big) \\
=& \frac{C}{r^*}M,
\end{aligned}
$$

where the constant $M \approx 1.7475$ is the approximate value of the alternating series, and is termed the Madelung constant after its discoverer.

(a) Compute the bonding energy of a more simplified (non-real) crystal consisting of a long chain of alternating Na^+-Cl^- atoms, each spaced r^* apart.

(b) Compute the Taylor series approximation to $\ln(2)$ centered at the point $a = 1$ and compare with (a).