

Name:

Score:

Math 1321      Week 2 Worksheet      Due Thursday 09/11

1. (5 points) Test for convergence or divergence. Cite any tests or theorems used in your solutions.

(a)  $\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{n^2+5}$

(b)  $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

(c)  $\sum_{n=1}^{\infty} \frac{n!}{3^{n-1}}$

(d)  $\sum_{n=0}^{\infty} \frac{(-1)^{n-3} \sqrt{n}}{n+4}$

(e)  $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$

2. Bond energy between salt ions  $\text{Na}^+$  and  $\text{Cl}^-$ . Positive and negative charges form an attractive force  $f_a$  proportional to the inverse square of the distance  $r$  between the charges, which can cause the sodium and chloride to combine in a solid crystal

$$f_a(r) = -\frac{A}{r^2},$$

where  $A$  is a positive constant and the negative sign  $-$  indicates that the force  $f_a$  pulls the two opposing charges together. For large distances  $r$ , the force is weak, but nearby the force gets stronger, much like the attractive force of a magnet. The two atoms also have a repulsive force that prevents the atoms from fusing infinitesimally close to each other.

$$f_b(r) = \frac{B}{r^n},$$

where  $B$  is a positive constant and the force goes with the inverse  $n$ th power of distance (the power is typically anywhere between the 7th and 13th power, depending on the particular atoms involved).

The net force  $f_{net}$  on the two ions is

$$f_{net}(r) = f_b(r) + f_a(r)$$

The ions are in a stationary equilibrium when their position  $r^*$  is such that the net force is zero:  $f_{net} = 0$ .

(a) **(1 points)** Solve for  $r^*$

(b) **(2 points)** From the equilibrium point, compute the bonding energy of the two molecules by computing the work involved to move the two ions from  $r^*$ , to a position infinitely far apart. That is, compute the work done against the force  $f_{net}(r)$  as we pull apart the ions by making  $r$  go to infinity to verify that  $W(r^*) = \frac{C}{r^*}$  where  $C$  is a positive constant that depends on  $n$  and  $A$ .