Math 2250 Lab 2   Name/Unid: _____

Due Date: 01/29/2015

1. Consider the set up consisting of two brine tanks where water cascades from tank 1 down into tank 2, and then out of tank 2. (see the following picture). Suppose that when the system is set into motion, the first tank contains 20 gallons of brine, while the tank below it contains 40 gallons of pure water. When the system starts, fresh water is pumped into tank 1 at a constant rate of 5 gal/min. At the same time the brine solution in tank 1 drains into tank 2 at a rate of 5 gal/min, and tank 2 drains at this same rate. Hence the volumes in both tanks stay constant. Assume that the solutions in each tank remain well-mixed.

Tank **1**
Volume V_1
Amount x

Tank **2**
Volume V_2
Amount y
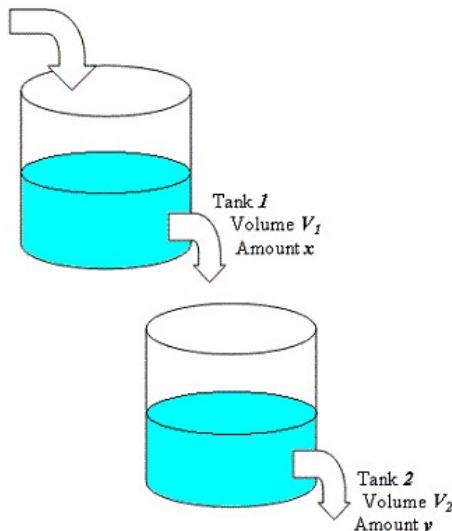
(a) If the first tank originally contains 24 lbs of salt, solve an initial value problem to find the amount $x(t)$ of salt in tank 1 at time t.

   1. First show that $x(t)$ satisfies the DE $\frac{dx}{dt} = -\frac{x}{4}$.
   2. Show details for find the solution to the above DE.

(b) Suppose that $y(t)$ is the amount of salt in tank 2 at time t. Show first that $\frac{dy}{dt} = \frac{x}{4} - \frac{y}{8}$. Then solve for $y(t)$ using the function $x(t)$ found in part (a).

<u>References</u>: Edwards & Penney Section 1.5

Solution:

(a) The concentration of salt in the first tank at any given time is given by $\frac{x(t)}{20}$ and so the rate at which salt leaves the tank is $5\frac{x(t)}{20}$. There is no salt coming into tank 1, so we have $\frac{dx}{dt} = 0 - 5\frac{x(t)}{20} = \frac{-x(t)}{4}$.

(b) The rate at which salt enters the second tank is just the rate it leaves the first, which is $\frac{x(t)}{4}$. The rate at which it leaves the second tank will be the rate at which the brine solution is leaving multiplied by the concentration of salt in the tank at any time, $5\frac{y(t)}{40} = \frac{y(t)}{8}$.

Thus $\frac{dy}{dt} = \frac{x(t)}{4} - \frac{y(t)}{8}$. Now substitute the answer from the first part to get $\frac{dy}{dt} = 6e^{\frac{-t}{4}} - \frac{y(t)}{8}$.

2. A 24-year-old woman accepts an engineering position with a starting salary of $50,000 per year. Her salary $S(t)$ increases exponentially at a continuous rate of 5% a year, so that $S(t) = 50e^{t/20}$ thousand dollars per year, after t years. Meanwhile, 12% of her salary is deposited continuously into a retirement account, which accumulates interest at a continuous annual rate of 4%. Let $A(t)$ be the amount in the retirement account after t years, with $A(0) = 0$ thousands of dollars at the time she begins her new job.

   (a) Recall discrete estimation of continuous rate: suppose there is an independent account $B(t)$ accumulating interest at an annual continuous rate of 4%, then estimate the change in the account balance $\Delta B(t)$ in terms of Δt.

   (b) Estimate ΔA in terms of Δt, to derive the differential equation satisfied by $A(t)$.

   (c) Compute $A(46)$, the amount available for her retirement at age 70.

   <u>References</u>: Edwards & Penney Section 1.5
   To learn more about different forms of interest rate, you can refer to: Wikipedia-http://en.wikipedia.org/wiki/Compound_interest
   Also, you can find relevant models in Professor Gustafso's online note:
   exponentialApplicationLibrary.pdf

Solution:

   (a) The change in the account balance $\Delta B(t)$ is the interest earned during Δt, which is 3%Δt of the account balance $B(t)$. Therefore, $\Delta B(t) = 0.04B(t)\Delta t$.

   (b) The retirement account accumulates money from compounding interest, and from the salary deposit. So, $\Delta A \approx (.04)A(t)\Delta t + (0.12)S(t)\Delta t$. Dividing by Δt and letting Δt go to 0, we get $\frac{dA}{dt} = .04A(t) + (0.12)50e^{.05t}$. Simplifying,

$$\frac{dA}{dt} = .04A(t) + 6e^{.05t}$$

   .

3. Match each differential equation with the corresponding vector field. For each slope field, sketch a few likely solution curves.

Differential equations:

1. $\dfrac{dy}{dx} = \dfrac{x-1}{y}$

2. $\dfrac{dy}{dx} = y - \sin x$

3. $\dfrac{dy}{dx} = x^2 + y^2 - 1$

4. $\dfrac{dy}{dx} = x^2 + 1$

Vector fields:

(a)

(c)

(b)

(d)
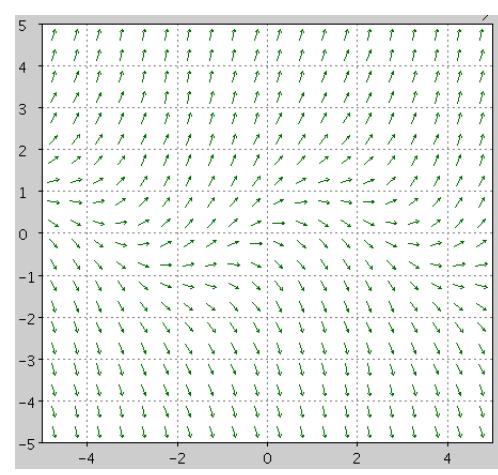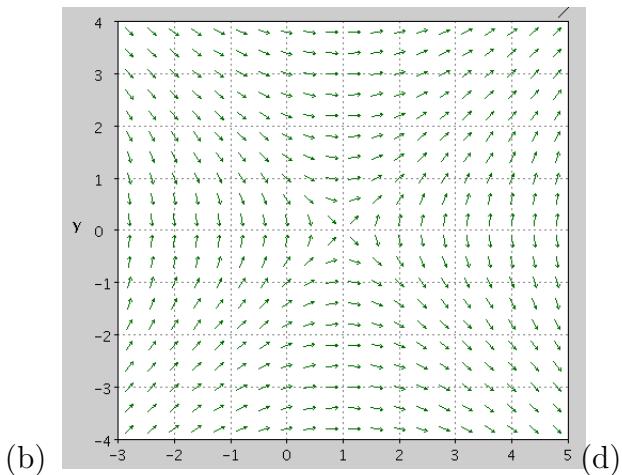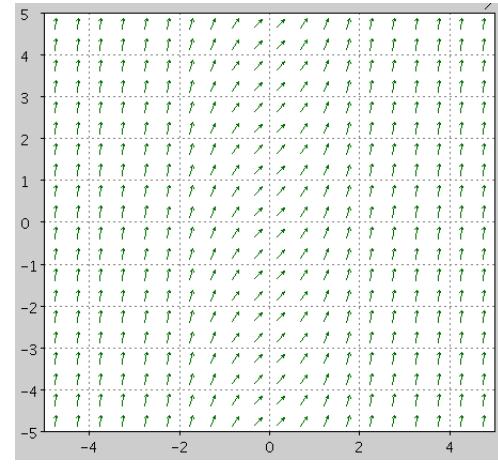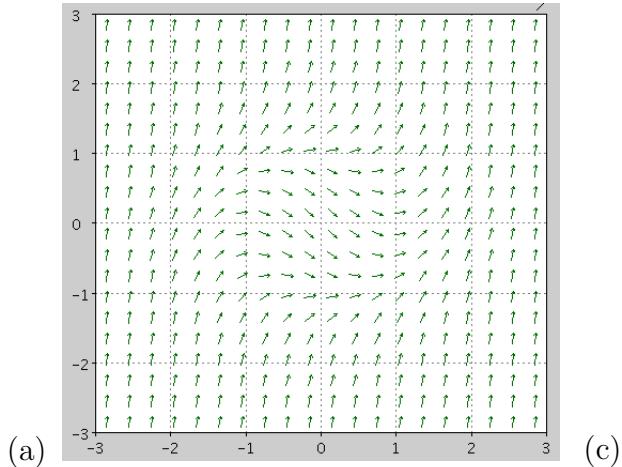
<u>References</u>: Edwards & Penney Section 1.3

Solution:

Vector fields:

(a)