代数基础 (2000590002501) 2025-2026 学年第一学期 课程大纲

教师: 朱子文

办公室: 宁静楼 313

电子邮箱: zzhu@tongji.edu.cn

时间地点: 周三 13:30pm-15:05pm, 彰武北大楼 312

答疑时间: 待定

课程主页: https://zw-zhu.github.io/Alg.html

教材: Algebra (Revised Third Edition), Serge Lang, Springer, ISBN 978-1-4612-6551-1.

学分: 本课程共计 4 学分.

References:

1. Algebra: Chapter 0, Paolo Aluffi, American Mathematical Society.

2. 代数基础:模、范畴、同调代数与层 (修订版), 陈志杰, 高等教育出版社.

Prerequisites: The audience is assumed to be familiar with linear algebra and undergraduate-level abstract algebra (e.g. basic group, ring and field theory). Additionally, a full knowledge of relevant undergraduate courses such as analysis and topology would also help.

Course Description: The course will cover basic notions about algebra which is widely used in many fields of mathematics. We will discuss topics related to category theory, modules, homological algebra and sheaf theory.

Course outline: The schedule of the course is as follows:

Week 1	Review of algebra, from categorical point of view
Week 2	Functors and natural transformation
Week 3	Fundamentals of modules
Week 4	Modules over PIDS
Week 5	Projective and injective modules
Week 6	introduction of commutative algebra
Week 7	Tensor products of modules, linear algebra

Week 8	Flat modules
Week 9	Abelian category
Week 10	Derived functor
Week 11	Ext and Tor
Week 12	Sheaf theory
Week 13	Sheaf cohomology
Week 14	Applications
Week 15	Selected advanced topics
Week 16	Selected advanced topics

This is an optimistic idea of how we will progress through the material. I reserve the right to change the schedule at will, in order to take into consideration the dynamics of the class.

Homework:

- Homework will be assigned featuring problems selected from the textbook.
- Some exercises will be assigned during the course. These exercises will not be graded, but they can help you get used to new concepts.
- 40% of your final score will be determined by your performance on the homework.
- 20% of your final score will be determined by a short essay briefly explaining one of the following topics: application of Yoneda lemma, Jordan form via finitely generated modules over PIDs, de Rham theorem, etc.

Final exam:

40% of your final score will be determined by your performance on the final exam. The final will be at the level of a typical qualifying exam on algebra for PhD candidates. More details about final exam will be announced toward the end of the semester.