Complex Geometry(210217601)
(Introduction to Birational Geometry)
2024 Fall

Instructor Information

Name: Ziwen Zhu
Office: 313 Ningjing Building
Email: zzhu@tongji.edu.cn
Office Hours: TBA
Website: https://zw-zhu.github.io

Class Information

Time: Th 13:30 – 16:15
Classroom: 104 Ningjing Building

Course Description

This course is an introduction to birational geometry. The aim is to introduce some of the core concepts and techniques in the birational classification theory of algebraic varieties. The audience is assumed to be familiar with basics of algebraic geometry, e.g. Hartshorne Chapter II and III or other equivalences.

Course Outlines

We will cover the following subjects:

- Basics notions in birational geometry: divisorial valuations, divisors, divisorial sheaves, canonical divisors, log resolutions, sigularities in MMP, intersection numbers, adjunction etc.
- Tools in birational geometry: bend and break, vanishing theorems, multiplier ideal sheaves;
- Introduction to MMP, related theorems: Cone Theorem, Base-point-free Theorem, Rationality Theorem, Nonvanishing Theorem, etc. MMP with scaling, PL flips
- Sketch of the inductive proof of BCHM.
- Immediate consequences of BCHM and applications.
- selected topics: K-stability, boundedness, moduli spaces of stable pairs, generalized pairs and complements.

References

The following textbooks are good sources to get used to common notations and arguments in the study of birational geometry and MMP:

- Birational geometry of algebraic varieties, János Kollár & Shigefumi Mori

- Positivity in algebraic geometry I & II, Robert Lazarsfeld

Most of the standard results in MMP can be found in the following papers, textbooks and notes:

- Existence of minimal models for varieties of log general type, Caucher Bikar, Paolo Cascini, Christopher Hacon & James M^{c}Kernan
- Classification of higher dimensional algebraic varieties, Christopher Hacon & Sándor Kovács
- Algebraic varieties: minimal models and finite generation, Yujiro Kawamata, translated by Chen Jiang
- Topic course notes on MMP, Christopher Hacon, https://www.math.utah.edu/∼hacon/MMP.pdf

Other related references:

- Complex algebraic surfaces, Arnaud Beauville
- Generalized divisors and reflexive sheaves on normal varieties, Karl Schwede, https://www.math.utah.edu/∼schwede/Notes/GeneralizedDivisors.pdf