微分流形 (210219001)　2024-2025 学年第一学期
课程大纲

教师: 朱子文
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时间地点: 周四 8:50am-11:35am, 宁静楼 117

答疑时间: 待定

课程主页: https://zw-zhu.github.io/Man.html

教材: *Introduction to Smooth Manifolds (Second edition)*, John Lee, Springer, ISBN 978-1-4419-9981-8.

学分: 本课程共计 3 学分.

References:

1. *An Introduction to Manifolds (Second Edition)*, Loring Tu, Springer.
2. *Foundations of Differentiable Manifolds and Lie Groups*, Frank Warner, Springer.
3. 微分几何讲义, 陈省身与陈维桓著, 北京大学出版社.

Prerequisites: The audience is assumed to be familiar with multivariable calculus, linear algebra and point-set topology. Additionally, knowledge about fundamentals of algebra such as groups, rings and modules would also help.

Course Description: The course will cover basic notions about differentiable manifolds so that we can do calculus on manifolds as on Euclidean spaces. The ultimate goal is to understand integration on manifolds and develop Stokes' theorem and De Rham theorem.

Course outline: We will cover the following subjects:

Week 1	Topological manifolds, smooth manifolds
Week 2	Examples, smooth maps
Week 3	Algebraic construction of manifolds, basic sheaf theory
Week 4	Tangent spaces, cotangent spaces, tangent bundles, cotangent bundles
Week 5	Vector bundles I and examples
Week 6	Submersions, immersions, embeddings
Week 7	Submanifolds, Sard's theorem

Week 8	Lie groups, vector fields, integral curves
Week 9	Distributions, Frobenius theorem, foliations
Week 10	Vector bundles II
Week 11	Differential forms
Week 12	Integration on manifolds
Week 13	Algebraic topology, Introduction/review
Week 14	De Rham cohomology
Week 15	De Rham theorem
Week 16	Examples and applications
Week 17	Selected topics: Riemannian metrics, complex manifolds, etc.

This is an optimistic idea of how we will progress through the material. I reserve the right to change the schedule at will, in order to take into consideration the dynamics of the class.

Homework:

- During the class, exercises will be assigned that involve verifying certain properties and explicit computation of examples.
- Additionally, homework will be assigned featuring problems selected from the textbook.
- 30% of your final score will be determined by your performance on the homework and exercises.

Final exam:

70% of your final score will be determined by your performance on the final exam. More details about final exam will be announced toward the end of the semester.